Nyaben Flight Dynamics

Analysis Results

FAA Level 5 and EASA FNPT II FSTD Qualification for Light Single-Piston Engine Aircraft

- Report for the Diamond DA40 -

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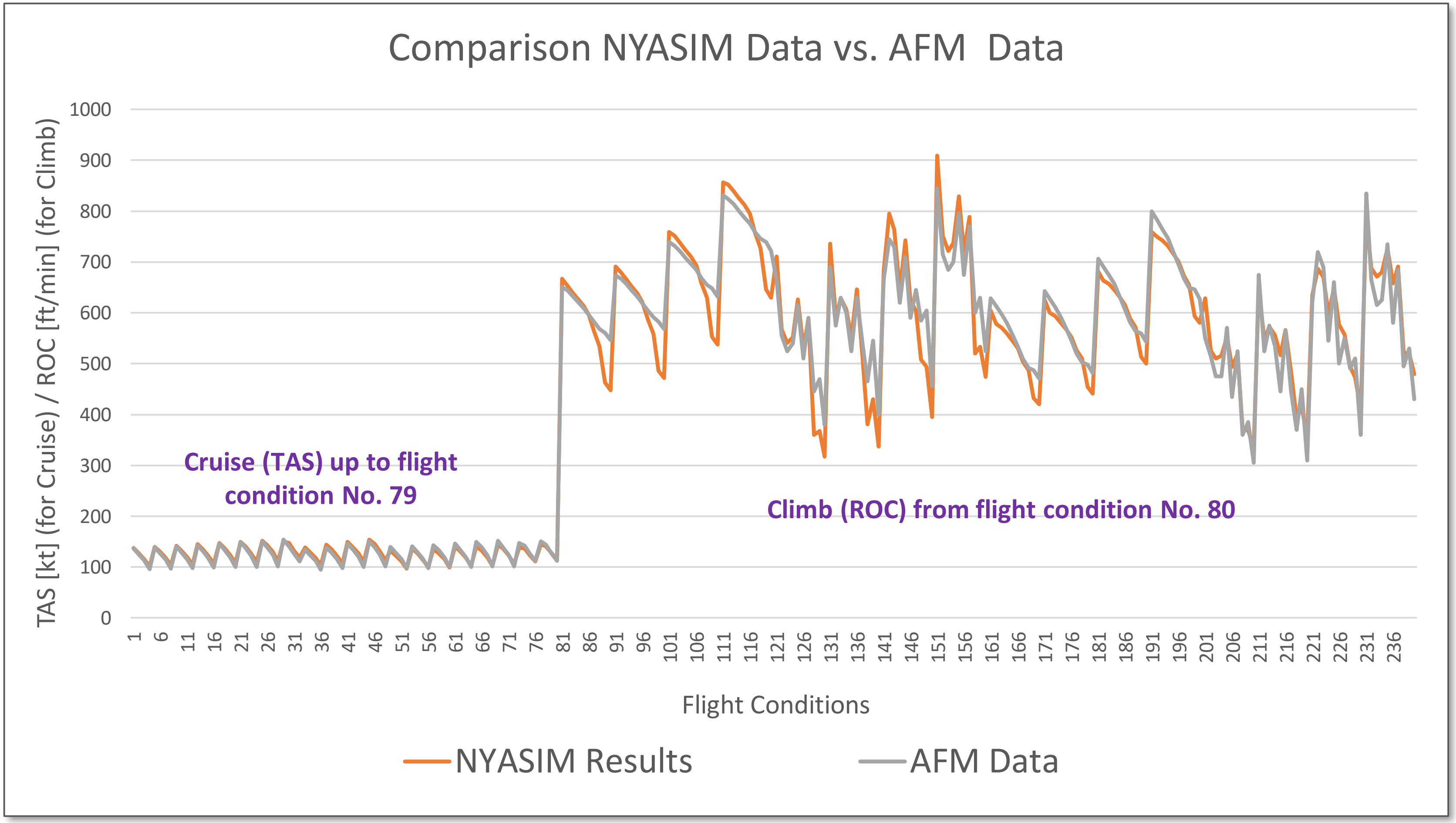
1.0 DA40 FDM (based on NYASIM) vs. AFM Data



3. Handling Qualities

The motion characteristics of all the airplanes except those of the DA40NFD are estimated based on the dimensionless derivatives as defined in some technical books and reports. If these derivatives and the corresponding flight conditions are known, the state space representation of the motion can be defined. Then the characteristic polynomials, the transfer functions and the eigenvalues can be estimated.

The derivatives for the aerodynamic model of the DA40NFD have firstly been calculated according to the approach described in [Ref. 13] by Jan Roskam. Afterwards they have been tuned in order to matched the level 1 requirements of the Cooper-Harper Rating scales.

For the linearization of the equations of motion the motion is decoupled into the longitudinal and lateral dynamics of the airplane. The following steps are made during the linearization:

- Choose Equilibrium Point

  The equilibrium point is the reference state where the airplane is trimmed out before the dynamic motion (the disturbed motion) is generated. The equilibrium point is a steady flight without any rotation of the airplane around its axes.

- Substitute

  All the variables in the dynamic motion are considered to be the sum of their values in the equilibrium point and a small part, which characterizes the dynamic changes (small-disturbance theory). The variables will be accordingly substituted in the equations of motion.

- Taylor Series

  After the substitution of the variables, the relationship between the forces, respectively the moments and the variables is transformed in Taylor series.

3. Handling Qualities

- Linearization

  In the Taylor series the nonlinear terms are excluded. The angles in the disturbed motion are considered as small. The linearization yields to the state-space representation for the longitudinal and lateral dynamics.

The state space representation has the following structure:

Where x is the state vector, u is the control vector, and the matrices A and B contain the aircraft's dimensional stability and control derivatives.

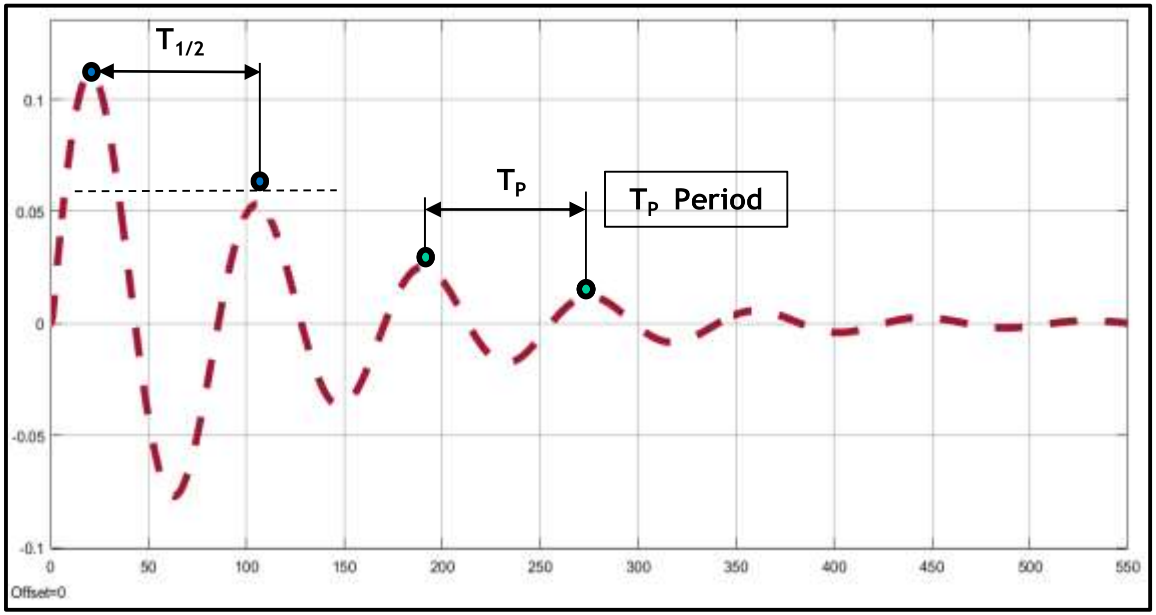
$$\begin{bmatrix}\Delta\dot{V}\\ \Delta\dot{\alpha}\\ \Delta\dot{q}\\ \Delta\dot{\Theta}\end{bmatrix}=\begin{bmatrix}X_V & X_\alpha & X_q & X_\Theta\\ Z_V & Z_\alpha & Z_q+1 & Z_\Theta\\ M'_V & M'_\alpha & M'_q+M_{\dot{\alpha}} & M_{\dot{\alpha}}Z_\Theta\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}\Delta V\\ \Delta\alpha\\ \Delta q\\ \Delta\Theta\end{bmatrix}+\begin{bmatrix}X_\eta & X_{\eta_K} & X_{\eta_F}\\ Z_\eta & Z_{\eta_K} & Z_{\eta_F}\\ M'_\eta & M'_{\eta_K} & M'_{\eta_F}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\eta\\ \eta_K\\ \eta_F\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 0 & -\frac{1}{V_r}\\ 0 & -M_{\dot{\alpha}}\frac{1}{V_r}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\dot{u}_{Ws}\\ \Delta\dot{w}_{Ws}\end{bmatrix}$$

State Space Representation of the Longitudinal Motion

$$\begin{bmatrix}\Delta\dot{\beta}\\ \Delta\ddot{\Phi}\\ \Delta\ddot{\Psi}\\ \Delta\dot{\Phi}\end{bmatrix}=\begin{bmatrix}Q'_\beta & Q'_p & Q'_r & Q'_\Phi\\ L'_\beta & L'_p & L'_r & 0\\ N'_\beta & N'_p & N'_r & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta\dot{\Phi}\\ \Delta\dot{\Psi}\\ \Delta\Phi\end{bmatrix}+\begin{bmatrix}Q'_\xi & Q'_\zeta\\ L'_\xi & L'_\zeta\\ N'_\xi & N'_\zeta\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\xi\\ \Delta\zeta\end{bmatrix}+\begin{bmatrix}Q'_{\dot{V}ws}\\ 0\\ 0\\ 0\end{bmatrix}\Delta\dot{v}_{Ws}$$

State Space Representation of the Lateral Motion

3. Handling Qualities: Characteristics



Time for halving the amplitude:

$$T_{1/2} = \frac{0{,}693}{|\sigma|}$$

Damped frenquency:

$$\omega = \frac{2 \cdot \pi}{T_P}$$

Natural frequency:

$$\omega_0 = \sqrt{\sigma^2 + \omega^2} = \sqrt{\left(\frac{0{,}693}{T_{1/2}}\right)^2 + \left(\frac{2 \cdot \pi}{T_P}\right)^2}$$

Damping ratio:

$$D = \frac{-\sigma}{\omega_0} = \frac{-0{,}693}{T_{1/2}} \cdot \sqrt{\left(\frac{0{,}693}{T_{1/2}}\right)^2 + \left(\frac{2 \cdot \pi}{T_P}\right)^2}$$

3.2 Analysis of the Phugoid, Short Period, Roll, Dutch Roll and Spiral Dynamics

	DA40 NFD
Altitude [ft]	12,000
TAS [m/s]	69.96
Wing area [m²]	13.54
Mass [kg]	1,213.9
Wing cord [m]	1.156
Wingspan [m]	12.08
I_X [kg m²]	3,781.29
I_Y [kg m²]	3,012.12
I_Z [kg m²]	6,130.67
I_{ZX} [kg m²]	0
α_{ref} [deg]	0
γ_{ref} [deg]	0
z_F [m]	0

Initial Conditions for the Motion in Cruise Configuration

3.2 Analysis of the Phugoid, Short Period, Roll, Dutch Roll and Spiral Dynamics

	DA40 NFD (Cruise Climb)
Altitude [ft]	6,000
TAS [m/s]	41.39
Wing area [m²]	13.54
Mass [kg]	1,277.4
Wing cord [m]	1.156
Wingspan [m]	12.08
I_X [kg m²]	4,800.68
I_Y [kg m²]	3,016.5
I_Z [kg m²]	7,151.26
I_{ZX} [kg m²]	0
α_{ref} [deg]	0
γ_{ref} [deg]	4.53
z_F [m]	0

Initial Conditions for the Motion in Climb Configuration

3.2 Analysis of the Phugoid, Short Period, Roll, Dutch Roll and Spiral Dynamics

	DA40 NFD
Altitude [ft]	3,000
TAS [m/s]	36.91
Wing area [m²]	13.54
Mass [kg]	1,120.5
Wing cord [m]	1.156
Wingspan [m]	12.08
I_X [kg m²]	2,308.06
I_Y [kg m²]	3,002.45
I_Z [kg m²]	4,652.79
I_{ZX} [kg m²]	0
α_{ref} [deg]	0
γ_{ref} [deg]	-3.0
z_F [m]	0

Initial Conditions for the Motion in Approach Configuration

3.2.1 *2.c.9* Phugoid Dynamics

3.2.1a Data for the Phugoid Dynamics in Climb, Cruise and Approach Configuration

PHUGOID ALL FLIGHT PHASES	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise Climb)	DA40 NFD (Cruise)	DA40 NFD (Approach)
Damping Ratio	$D_{Ph} \geq 0.04$	0.0051	0.1005	0.09
Period [s]	-	54.8755	56.9081	35.6512
Natural Frequency [Hz]	-	0.1145	0.111	0.177
Damped Frequency [Hz]	-	0.1145	0.1104	0.1762
Decay Coefficient	-	-5.8755e-04	-0.0111	-0.0159
Damping Angle [deg]	-	0.294	5.7663	5.163
Logarithmic Decrement	-	0.0322	0.6345	0.5677
Decay Time [s]	-	1.1797e+03	62.169	43.5276

3.2.2 *2.c.10* Short Period Dynamics

3.2.2a Data for the Short Period Dynamics (SP) in Cruise Configuration

SHORT PERIOD CRUISE	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise)
Damping Ratio	0.30 ≤ D ≤ 2.0	0.6974
Period [s]	-	1.4195
Natural Frequency [Hz]	-	6.1758
Damped Frequency [Hz]	-	4.4264
Decay Coefficient	-	-4.3068
Damping Angle [deg]	-	44.2155
Logarithmic Decrement	-	6.1134
Decay Time [s]	-	0.1609

3.2.2 *2.c.10* Short Period Dynamics

3.2.2b Data for the Short Period Dynamics (SP) in Climb and Approach Configuration

SHORT PERIOD CLIMB and APPROACH	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise Climb)	DA40 NFD (Approach)
Damping Ratio	0.35 ≤ D ≤ 1.30	0.7361	0.7618
Period [s]	-	2.2502	2.4017
Natural Frequency [Hz]	-	4.1251	4.0382
Damped Frequency [Hz]	-	2.7923	2.6162
Decay Coefficient	-	-3.0364	-3.0762
Damping Angle [deg]	-	47.398	49.6198
Logarithmic Decrement	-	6.8324	7.3879
Decay Time [s]	-	0.2283	0.2253

3.2.3 *2.d.2* Roll Response (rate)

3.2.3a Data for the Roll Dynamics (R) in Cruise Configuration

ROLL DYNAMICS CRUISE	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise)
Time Constant [s]	τ ≤ 1.4 s	0.4080
Damping Ratio	-	1.00
Natural Frequency [Hz]	-	2.6523
Damped Frequency [Hz]	-	0.00
Decay Coefficient	-	-2.6523
Damping Angle [deg]	-	90
Decay Time [s]	-	0.2613

Time constant for the roll motion: $\tau = \frac{-1}{L'_P}$

With $$L'_P = \frac{I_Z}{I_X \cdot I_Z - I_{XZ}^2} \cdot \frac{\rho}{2} V^2 \cdot S \cdot \frac{b}{2} \cdot \frac{b}{2 \cdot V} \cdot \left[C_{lp} + \frac{2 \cdot I_{XZ}}{I_Z \cdot b} \cdot C_{np} + \left(\frac{I_X}{I_Z} \cdot C_{np} + \frac{I_{XZ}}{I_Z} \cdot C_{lp} \right) \cdot \gamma - \left(C_{lr} + \frac{I_{XZ}}{I_Z} \cdot C_{nr} \right) \cdot \alpha \right]$$

3.2.3 *2.d.2* Roll Response (rate)

3.2.3b Data for the Roll Dynamics (R) in Climb and Approach Configuration

ROLL DYNAMICS CLIMB and APPROACH	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise Climb)	DA40 NFD (Approach)
Time Constant [s]	$\tau \leq 1.0$ s	0.7218	0.4341
Damping Ratio	-	1.00	1.00
Natural Frequency [Hz]	-	1.5709	2.3636
Damped Frequency [Hz]	-	0.00	0.00
Decay Coefficient	-	-1.5709	-2.3636
Damping Angle [deg]	-	90.00	90.00
Decay Time [s]	-	0.4412	0.2933

3.2.4 *2.d.7* Dutch Roll

3.2.4a Data for the Dutch Roll Dynamics (DR) in Cruise Configuration

DUTCH ROLL CRUISE	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise)
Damping Ratio	$D_{DR} \geq 0.08$	0.115
Decay Coefficient	$-\sigma \geq 0.15$	-0.2014
Natural Frequency [Hz]	$\omega_0 \geq 0.4$ Hz	1.7509
Damped Frequency [Hz]	-	1.7392
Period [s]	-	3.6126
Damping Angle [deg]	-	6.6057
Logarithmic Decrement	-	0.7276
Decay Time [s]	-	3.4414

3.2.4 *2.d.7* Dutch Roll

3.2.4b Data for the Dutch Roll Dynamics (DR) in Climb and Approach Configuration

DUTCH ROLL CLIMB and APPROACH	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise Climb)	DA40 NFD (Approach)
Damping Ratio	$D_{DR} \geq 0.08$	0.0868	0.2366
Decay Coefficient	$-\sigma \geq 0.15$	-0.0949	-0.2743
Natural Frequency [Hz]	$\omega_0 \geq 1.00$ Hz	1.0937	1.1594
Damped Frequency [Hz]	-	1.0895	1.1265
Period [s]	-	5.7669	5.5774
Damping Angle [deg]	-	4.9806	13.6838
Logarithmic Decrement	-	0.5476	1.5298
Decay Time [s]	-	7.3001	2.5271

3.2.5 *2.d.4* Spiral Stability

3.2.5a Data for the Spiral Dynamics (Spi) in all Configuration

SPIRAL STABILITY ALL FLIGHT PHASES	Cooper-Harper Level 1 Light AC	DA40 NFD (Cruise Climb)	DA40 NFD (Cruise)	DA40 NFD (Approach)
Decay Time [s]	$T_2 \geq 20$ s	47.1964	33.3816	(-)34.7524
Damping Ratio	-	1.00	1.00	-1.00
Natural Frequency [Hz]	-	0.0147	0.0208	0.0199
Damped Frequency [Hz]	-	0.00	0.00	0.00
Decay Coefficient	-	-0.0147	-0.0208	0.0199
Damping Angle [deg]	-	90.00	90.00	-90

The negative sign for the times indicates instability. This sign is in brackets because there are no negative times.

3.3 Plots of the QTG Test Results

See enclosed documents in the folder:

Plot Deliverables